# The Intrusive PCE-Based Method for Uncertainty Calculation in Ray-Tracing Analysis of 5G EM Wave Propagation

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***Abstract*—In the paper it is presented the novel intrusive method for simulation of stochastic electromagnetic (EM) fields for a 5G frequency band. This cannot be obtained using nowadays available non-intrusive general Polynomial Chaos (gPC) methods because they are not efficient when used for prediction of stochastic properties of millimeter EM waves. The novel intrusive gPC method enables to include the material, as well as geometrical, uncertainties in stochastic EM fields simulations. The novel method is applied to analysis of 5G stochastic EM fields propagation for the case of an outdoor-to-indoor scenario.**

***Index Terms*—millimeter wave propagation, random variables, polynomial chaos expansion**

## I. Introduction

This paper deals with uncertainty calculation associated with the propagation of random EM fields in 5G millimeter wave band. The most popular methods for simulation of propagation of random EM fields are the Monte Carlo method, e.g. [1], and general polynomial chaos (gPC) [2]. The gPC method is much more efficient than the Monte Carlo method for many applications in antennas and propagation area what was presented in many scientific papers, e.g. [3, 4]. It is associated mainly with the number of realizations of random variables which must be considered in order to find the moments of probability density functions (pdf) of an investigated stochastic EM field distribution. It is much smaller for the case of the gPC method than for the Monte Carlo method. The gPC method uses polynomials which are orthogonal with respect to a given joint probability density function of random parameters of a propagation scenario. These polynomials are used to obtain the Polynomial Chaos Expansion (PCE) of a considered random EM field. The coefficients of its PCE expansion are used then to derive moments as well as pdf of this random EM field. The method originates from the work of Norbert Wiener in 1938 [5] and since then was discussed in numerous articles and books, e.g. [6, 7]. The methods that implement the gPC theory can be distinguished into two types, non-intrusive and intrusive. The first type enables the direct application of deterministic EM fields solvers. The latter requires to modify the ordinary EM field solver, e.g., FDTD update equation as in[8].

The gPC method, to the best knowledge of the author, was not addressed to a simulation of propagation of random millimeter waves. The non-intrusive type of this method appears to be not efficient when the frequency of random EM field is about 5GHz and above. Monte Carlo method becomes much more attractive for such scenarios. The geometrical uncertainties associated with millimeter EM fields give rise to very long times of non-intrusive calculation of PCE meta-models. It is a consequence of a very fast variation of millimeter EM fields in a spatial domain. The details of commonly used in literature non-intrusive methods can be found in, e.g. [9].

The paper describes the novel intrusive method for simulation of random functions of electromagnetic fields, eventually the square of an amplitude of an electric field, for the case of millimeter waves. The method enables fast calculation of accurate polynomial chaos expansion (PCE) meta-models [9] of considered random millimeter EM field by using expanded ray transfer functions (TFs) associated with ray-tracing simulation. The proposed intrusive method enables to include the material, as well as geometrical, uncertainties in stochastic EM fields simulations. The latter relates, e.g., to a position of an EM wave source or an observation point. The accuracy of the proposed intrusive method is not affected by the fast variation of millimeter EM fields in a spatial domain. The novel method is applied to the exemplary outdoor-to-indoor scenario of propagation of random 5G EM wave for frequencies 38GHz and 60GHz. The efficiency of the new method is compared to the reference Monte Carlo method.

The proposed intrusive method enables the inclusion of stochastic antenna radiation pattern or stochastic plane wave amplitude and/or angle of incidence in a simulation of an outdoor-to-indoor scenario. In also enables fast analytical

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update of the PCE meta-model, e.g., when it is necessary to update nominal values of random variables. The derived with the proposed method PCE meta-models are used to analyze the distributions of percentiles of a square of an electric field amplitude with focus on dominant rays for an exemplary propagation scenario.

## II. The novel gPC method for 5G EM fields analysis

### A. The block diagram of the novel method

The calculation of PCE meta-model of a considered random function of an electric field at a given stochastic observation point is performed according to the diagram shown in Fig. 1.

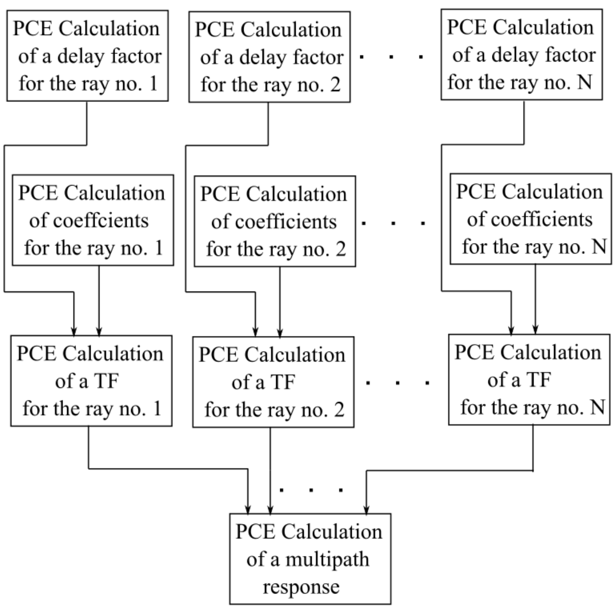


Fig. 1. The block diagram of the novel approach to simulation of stochastic EM wave propagation.

The PCE meta-model associated with a single ray is calculated in two steps. In the first step, the separate PCE meta-models related to delay and non-delay factors are calculated. The delay factors are exponential functions of an electric field phase. The non-delay factors include, e.g., diffraction and reflection coefficients. They can also include antennas stochastic transfer function. The degree of delay PCE meta-models is much higher than the degree of PCE non-delay meta-models because of the rapid spatial variation.

In the second step, the spectral projection is used to obtain the PCE meta-models of every single ray. At the beginning it is performed a product of a meta-model of a delay factor and a meta-model of a non-delay factor for each ray. Consequently, the non-orthogonal polynomials set approximates the transfer function associated with a single ray. The reason for this fact are products of polynomials of the same random variables. These variables correspond to spatial parameters which influence delay factors as well as non-delay factors. In order to obtain a PCE meta-model for each ray, the product of polynomials of the same variables is rearranged into the sum of orthogonal polynomials what will be presented in the next subsection.

In the last step of the novel method, the PCE meta-models of all considered rays are properly collected to derive the final PCE meta-model of an EM field.

### B. PCE meta-model for a single ray

As was described in the previous subsection, the PCE meta-model of a single ray is obtained using the PCE meta-models of a delay and non-delay factor corresponding to this ray. The latter can be obtained using an intrusive algorithm, e.g., a Matlab-based package called UQLab [9]. The PCE meta-model of a delay factor for ray no. $n$ and pulsation sample $\omega_s$ can be expressed as:

$$HD_n(\omega_s, \boldsymbol{\xi_{sp}}) \approx \sum_{\boldsymbol{j}} AHD_{n,\boldsymbol{j}} \cdot \Psi D_{n,\boldsymbol{j}}(\boldsymbol{\xi_{sp}}) \tag{1}$$

where: $\boldsymbol{\xi_{sp}}$ is the vector of spatial random variables, $\boldsymbol{j}$ is a multi-index [2] and $AHD_{n,\boldsymbol{j}}$ is a PCE coefficient corresponding to orthogonal polynomial $\Psi D_{n,\boldsymbol{j}}(\boldsymbol{\xi_{sp}})$. Multi-index $\boldsymbol{j}$ is expressed as $\boldsymbol{j} = \{d_0, d_1, d_2\}$ for three spatial coordinates.

The analogous PCE meta-model of a non-delay factor can be formulated as follows:

$$\begin{aligned} &HC_n(\omega_s, \boldsymbol{\xi_{sp}}, \boldsymbol{\xi_{mat}}) \approx \\ &\sum_{\boldsymbol{k}} AHC_{n,\boldsymbol{k}} \cdot \Psi C_{n,\boldsymbol{k}}(\boldsymbol{\xi_{sp}}, \boldsymbol{\xi_{mat}}) \end{aligned} \tag{2}$$

where: $\boldsymbol{\xi_{mat}}$ is the vector of material random variables (permittivities, conductivities of walls or faces of wedges), $\boldsymbol{k}$ is a multi-index and $AHC_{n,\boldsymbol{k}}$ is a PCE coefficient corresponding to orthogonal polynomial $\Psi C_{n,\boldsymbol{k}}(\boldsymbol{\xi_{sp}}, \boldsymbol{\xi_{mat}})$. Multi-index $\boldsymbol{k}$ can e written as $\boldsymbol{k} = \{d_0, d_1, d_2, c_0, c_1, ..., c_{m-1}\}$, where $m$ is the number of random material parameters which influence the non-delay factor of the $n-th$ ray.

When expressions (1) and (2) are multiplied the following non-orthogonal approximation is obtained:

$$H_n(\omega_s, \boldsymbol{\xi_{sp}}, \boldsymbol{\xi_{mat}}) \approx \sum_{\boldsymbol{j}} \sum_{\boldsymbol{k}} A_{n,\boldsymbol{j},\boldsymbol{k}} \Psi_{n,\boldsymbol{j},\boldsymbol{k}}(\boldsymbol{\xi_{sp}}, \boldsymbol{\xi_{mat}}) \tag{3}$$

where:

$$A_{n,\boldsymbol{j},\boldsymbol{k}} = AHD_{n,\boldsymbol{j}} \cdot AHC_{n,\boldsymbol{k}} \tag{4}$$

$$\Psi_{n,\boldsymbol{j},\boldsymbol{k}}(\boldsymbol{\xi_{sp}}, \boldsymbol{\xi_{mat}}) = \Psi D_{n,\boldsymbol{j}}(\boldsymbol{\xi_{sp}}) \cdot \Psi C_{n,\boldsymbol{k}}(\boldsymbol{\xi_{sp}}, \boldsymbol{\xi_{mat}}) \tag{5}$$

Vectors of random variables $\boldsymbol{\xi_{sp}}$ and $\boldsymbol{\xi_{mat}}$ are independent. Consequently, polynomial $\Psi C_{n,\boldsymbol{k}}(\boldsymbol{\xi_{sp}}, \boldsymbol{\xi_{mat}})$ can be expressed as the following product:

$$\Psi C_{n,\boldsymbol{k}}(\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{mat}}) = \Psi Cs_{n,\{d_0,d_1,d_2\}}(\boldsymbol{\xi_{sp}}) \cdot \Psi Cm_{n,\{c_0,c_1,\ldots,c_{m-1}\}}(\boldsymbol{\xi_{mat}}) \tag{6}$$

Polynomials $\Psi D_{n,\boldsymbol{j}}(\boldsymbol{\xi_{sp}})$ are orthogonal for each pair of indexes $\boldsymbol{j}$ and $\Psi C_{n,\boldsymbol{k}}(\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{mat}})$ are orthogonal for each pair of indexes $\boldsymbol{k}$, however, polynomials $\Psi_{n,\boldsymbol{j},\boldsymbol{k}}(\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{mat}})$ are not orthogonal what was described in the previous subsection. In order to transform (3) into a PCE expansion, the following substitution is applied:

$$\begin{aligned}
&\Psi D_{n,\{d_{00},d_{01},d_{02}\}}(\boldsymbol{\xi_{sp}}) \cdot \Psi Cs_{n,\{d_{10},d_{11},d_{12}\}}(\boldsymbol{\xi_{sp}}) = \\
&\prod_{i=0}^{2} \varphi_{n,d_{0i}}(\xi_{spi}) \prod_{i=0}^{2} \varphi_{n,d_{1i}}(\xi_{spi}) = \\
&\prod_{i=0}^{2} \sum_{d_{2i}=0}^{d_{2i}=d_{0i}+d_{1i}} B_{n,d_{2i}} \cdot \varphi_{n,d_{2i}}(\xi_{spi}) = \\
&\sum_{\{d_{20},d_{21},d_{22}\}} \prod_{i=0}^{2} B_{n,d_{2i}} \varphi_{n,d_{2i}}(\xi_{spi}) = \\
&\sum_{\boldsymbol{p}} BS_{n,\boldsymbol{p}} \Psi S_{n,\boldsymbol{p}}(\boldsymbol{\xi_{sp}})
\end{aligned} \tag{7}$$

where: multi-index $\boldsymbol{p}$ is equal to $\{d_{20},d_{21},d_{22}\}$, $\boldsymbol{\xi_{sp}} = \{\xi_{sp0},\xi_{sp1},\xi_{sp2}\}$ is a vector of spatial random variables and $B_{n,d_{2i}}$ are derived in the process of expansion of the product of univariate polynomials $\varphi_{n,d_{0i}}(\xi_{spi})$ and $\varphi_{n,d_{1i}}(\xi_{spi})$ into the sum of the same class of univariate polynomials (e.g., Hermite, Jacobi polynomials) of maximum order $d_{0i}+d_{1i}$ [2].

When (7) is substituted to (5) expression (3) takes the following form:

$$H_n(\omega_s,\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{mat}}) \approx \sum_{\boldsymbol{p},\boldsymbol{q}} AP_{n,\boldsymbol{p},\boldsymbol{q}} \Psi P_{n,\boldsymbol{p},\boldsymbol{q}}(\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{mat}}) \tag{8}$$

where: $\boldsymbol{q} = \{c_0,c_1,...,c_{m-1}\}$, while:

$$\Psi P_{n,\boldsymbol{p},\boldsymbol{q}}(\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{mat}}) = \Psi S_{n,\boldsymbol{p}}(\boldsymbol{\xi_{sp}}) \cdot \Psi Cm_{n,\boldsymbol{q}}(\boldsymbol{\xi_{mat}}) \tag{9}$$

and $AP_{n,\boldsymbol{p},\boldsymbol{q}}$ depends on $BS_{n,\boldsymbol{p}}$ and $A_{n,\boldsymbol{j},\boldsymbol{k}}$.

### C. PCE meta-model for all rays

The vector of random variables $\boldsymbol{\xi_{mat}}$ is not in general the same for all the rays included in the simulation of EM wave propagation and can be denoted by $\boldsymbol{\xi_{n,mat}}$. Consequently, the sum of transfer functions (8) of all the rays included in the propagation scenario is given as follows:

$$H(\omega_s,\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{mat}}) \approx \sum_{n} H_n(\omega_s,\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{n,mat}}) \tag{10}$$

Expressions $H_n(\omega_s,\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{n,mat}})$ which depend on the same vector of random variables need to be collected. Consequently, (10) can be rearranged into the following form of PCE meta-model of a transfer function of all the considered in simulation rays:

$$H(\omega_s,\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{mat}}) \approx \sum_{r} H_r(\omega_s,\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{r,mat}}) \tag{11}$$

where:

$$H_r(\omega_s,\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{r,mat}}) \approx \sum_{\boldsymbol{p},\boldsymbol{q}} AP_{r,\boldsymbol{p},\boldsymbol{q}} \Psi P_{r,\boldsymbol{p},\boldsymbol{q}}(\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{r,mat}}) \tag{12}$$

where $AP_{r,\boldsymbol{p},\boldsymbol{q}}$ is a sum of coefficients $AP_{n,\boldsymbol{p},\boldsymbol{q}}$ associated with the same vector of random variables $\boldsymbol{\xi_{n,mat}}$.

### D. PCE meta-model for a square of an electric field amplitude

The PCE meta-model in (12) does not enable direct calculation of a PCE meta-model of a square of magnitude of the rays transfer function. In order to do this expression (12) need to be decomposed into real and imaginary parts:

$$\begin{aligned}
&Re\left\{H_r(\omega_s,\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{r,mat}})\right\} \approx \\
&\sum_{\boldsymbol{p},\boldsymbol{q}} Re\left\{AP_{r,\boldsymbol{p},\boldsymbol{q}}\right\} \Psi P_{r,\boldsymbol{p},\boldsymbol{q}}(\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{r,mat}})
\end{aligned} \tag{13}$$

$$\begin{aligned}
&Im\left\{H_r(\omega_s,\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{r,mat}})\right\} \approx \\
&\sum_{\boldsymbol{p},\boldsymbol{q}} Im\left\{AP_{r,\boldsymbol{p},\boldsymbol{q}}\right\} \Psi P_{r,\boldsymbol{p},\boldsymbol{q}}(\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{r,mat}})
\end{aligned} \tag{14}$$

The square of a magnitude of the rays transfer function can be now written as follows:

$$\begin{aligned}
&\left\{Mag\left\{H(\omega_s,\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{mat}})\right\}\right\}^2 = \\
&\left\{\sum_{r} Re\left\{H_r(\omega_s,\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{r,mat}})\right\}\right\}^2 + \\
&\left\{\sum_{r} Im\left\{H_r(\omega_s,\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{r,mat}})\right\}\right\}^2
\end{aligned} \tag{15}$$

Using manipulations on polynomials as in (7) the PCE meta-model of (15) can be formulated using the new multi-index:

$$\begin{aligned}
&\left\{Mag\left\{H(\omega_s,\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{mat}})\right\}\right\}^2 = \\
&\sum_{\boldsymbol{u}} AU_u \Psi U_{\boldsymbol{u}}(\boldsymbol{\xi_{sp}},\boldsymbol{\xi_{u,mat}})
\end{aligned} \tag{16}$$

It should be noted here that each vector $\xi_{\boldsymbol{u},mat}$ is a subset of the vector $\xi_{mat}$.

section by applying the product of a PCE meta-model of the antenna transfer function and (16). However, it is not considered in the simulation example. The assumed random variables correspond to the position of an observation point. They have uniform distributions. It is observed that geometrical uncertainties have a greater influence on the requested random electric field than the random material parameters as in [6]. The number of PCE coefficients (1) is 26 when the EM wave frequency is 38GHz, while it is 40 for 60GHz. The number of PCE coefficients in (2) is 12 for both frequencies. The approximation errors are below 0.5%. The number or realizations used for Monte Carlo simulations were 12000 and 15000 ro 38GHz and 60GHz, respectively. The relative permittivity and conductivity of all walls are assumed to be 8 and 0.5S/m, respectively. The relative permittivity of glass is assumed to be 5. The 3D diffraction coefficients are calculated as in [11]. The aim of the simulation is to compare the simulation results obtained using the novel method with the results obtained using the reference Monte Carlo method in terms of accuracy and time of simulations. The results of simulations for frequencies 38GHz and 60GHz are shown in Fig. 3 and Fig. 4, respectively. The results obtained using the novel method are indicated by cross signs. The Monte Carlo results are indicated by circle graphs.

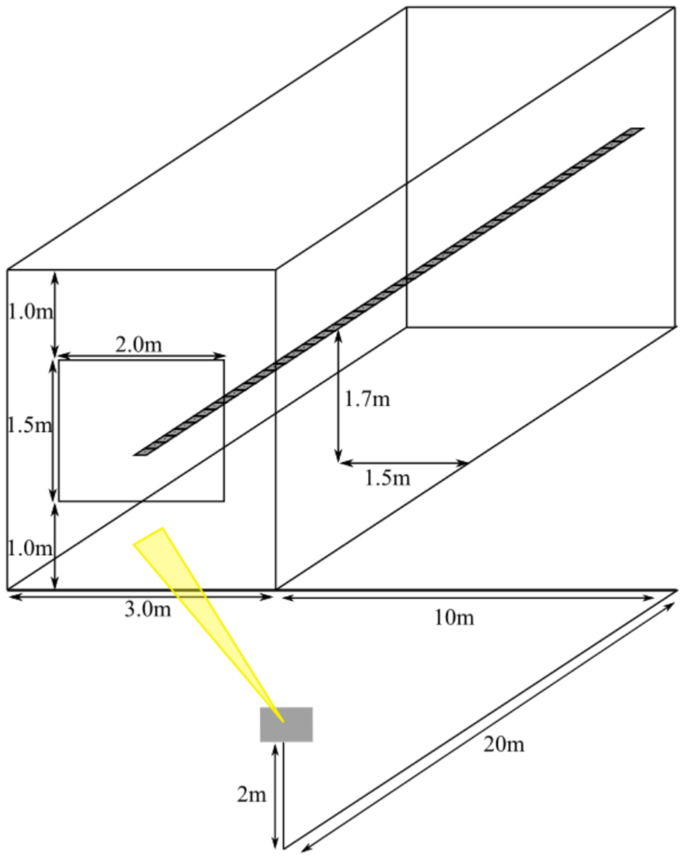


Fig. 2. The scenario of the simulation example.

## III. Simulation example

To verify the method for calculation of PCE meta-models associated with the simulation of random EM waves in the millimeter band, the exemplary outdoor-to-indoor scenario shown in Fig. 2 is analyzed. The width and height of the window are 2.0m and 1.5m, respectively. The observation path is 1.7m above the floor of a corridor, as it is shown in Fig. 2. The observation path is 10m long and is divided into $0.2\text{x}0.2m$ uncertainty squares. The nominal positions of observation points are in the middle of uncertainty squares. The observation path is 1m away from the window. It is assumed that the transmitter antenna is modeled by a directional radiation pattern. This radiation pattern has 30 degrees half power angle in nominal azimuth and elevation planes. The receiver antenna is omnidirectional. Transmitter and receiver antennas have vertical polarization. The results of the simulations are the 10th percentiles of spatial attenuation of a square of an electric field amplitude along the observation path for optimal direction of the transmitter antenna beam. The percentiles are calculated using the maximum entropy principle [10]. The position of the transmitter antenna is assumed to be deterministic. The shape of the radiation pattern of a transmitter antenna can be described by a vector of random variables and added with no significant effort to those already introduced in the previous

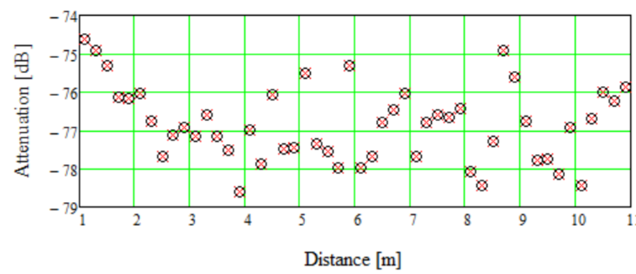


Fig. 3. The 10th percentile of the attenuation of EM wave along the observation path in Fig. 2 for frequency 38GHz.

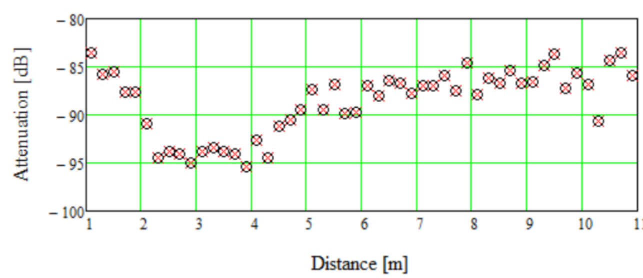


Fig. 4. The 10th percentile of the attenuation of EM wave along the observation path in Fig. 2 for frequency 60GHz.

The times of calculations were 4.58 minutes and 7.24 minutes for the case of the proposed novel method for frequencies 38GHz and 60GHz, respectively. The corresponding simulation times of Monte Carlo simulations were 171.38 minutes 259.64 minutes. It is evident that the novel method enables to achieve a great reduction of the described simulation times compared to the Monte Carlo method. The result of simulations obtained using both methods are in a great agreement what can be seen in Fig. 3 and in Fig. 4.

## IV. Conclusions

In the paper, it was presented the novel method for calculation of stochastic parameters of random EM fields for millimeter wave bands. It implements general polynomial chaos and its polynomial chaos expansion (PCE meta-models). From the experience of the author nowadays non-intrusive PCE methods cannot deal efficiently with this problem due to the very fast variation of millimeter EM fields in a spatial domain. The Monte Carlo method is a better choice for propagation scenarios at these frequencies. The method novel can implement uncertainties originating from the propagation channel as well as antennas. The simulation example presented in the previous section indicates a very high accuracy of the simulation results obtained using the novel method while it enables evident reduction of simulation times compared to the Monte Carlo method. More complex propagation scenarios are planned to be investigated using the presented in the paper method. These would include investigation of antennas uncertainties and complex sensitivity analysis.

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