Step Response Sensitivity to RLC Parameters of VLSI Interconnect

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***Abstract*— In the paper there is considered sensitivity of the voltage step response of the system inverter-interconnect-inverter with respect to RLC parameters. The sensitivity formulas for RLC parameters are given. There is also analyzed voltage step response deviation to relative parameter RLC variations.**

I. Introduction

The faster circuits mean that the impact of interconnects becomes more significant, and there is no possibility to neglect them in VLSI system analysis. Then we need some tools to interconnect systems examination. Some research are devoted to RLC parameter extraction to built the model of single or coupled interconnect. There are a lot of works about simulation and modeling the coupled interconnects. But there are some papers dedicated to single interconnect too [1]. The simulation of single interconnect allows to predict the crosstalk, time delay in the interconnect and can be the first step in VLSI system analyze. Then simulation and modeling the VLSI inverter-interconnect-inverter system is an important task in VLSI expansion.

The higher level interconnect must be modeled as RLC transmission line, as the inductance of such line is relatively big compared to resistance. The output step response for such model of interconnect can be obtain heuristically using the moment matching method [2], but can be also derived by multiple scales method [3]. After some transformation there is also possible to calculate the threshold crossing time for the system inverter-interconnect-inverter. The closed form formula of the threshold crossing time can be very helpful in time delay of interconnect system estimation.

The influence of the parameters change to system functionality can be expressed by sensitivity. It allows to single out the important parameters which have the most important impact on the system. It can be also used to find the parameters which can be neglected in system analysis. The sensitivity calculations of VLSI system parameters can be found e.g. in [4,5]. But there is a problem to calculate the sensitivity if there is no exact formula for the system response.

In this work we present the sensitivity of step response analysis for system inverter-interconnect-inverter. The sensitivity to RLC parameters are considered and some general features for sensitivity are presented.

The work is organized as follows. In next section there are presented the foundations of step response calculation. In third section there are sensitivity for RLC parameters formulas presented. In the fourth section there is analysis of sensitivity presented and some examples can be done. In the last section the conclusions are done.

II. Step response formulation

The step response calculation for higher level interconnects must be done assuming RLC transmission model. Furthermore we assume that global resistance is less than lossless interconnect impedance according to [6]. We will present only an outline of the method, which can be found in [2]. First for the low resistance interconnect we can write:

$$\frac{R_t}{2Z_0} \le 1 \qquad (1)$$

assumption allows treat the line as low-loss line with high inductance influence. The considered system is shown in Fig.1.

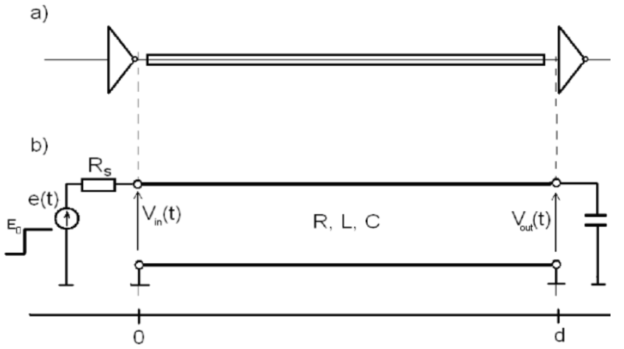

Fig. 1. a) The considered system inverter-interconnect-inverter, b) the model of the considered system

For the system of differential equation (2) with initial and boundary conditions (3,4) we could use the perturbation method of multiple scales, using the $\varepsilon = R_t / Z_0$ as perturbation parameter ($R_t = R \cdot d$).

$$\begin{cases} -\dfrac{\partial v}{\partial x} = Ri + L\dfrac{\partial i}{\partial t}, \\ -\dfrac{\partial i}{\partial x} = C\dfrac{\partial v}{\partial t} \end{cases} \qquad (2)$$

$$i(x,0) = 0, \quad v(x,0) = 0, \qquad (3)$$

$$
\begin{aligned}
&e(t) - R_s i(0,t) = v(0,t), \\
&-i(d,t) = C_0 \frac{\partial v(d,t)}{\partial t},
\end{aligned} \tag{4}
$$

where R, L, C – line parameters, C_0 – input inverter capacitance, R_s – output inverter resistance, $i(x,t)$, $v(x,t)$ – current and voltage in line, respectively, d – line length, t, x – time and space variable, respectively.

After scaling system (2) to obtain the perturbation parameter in the system, we can observe, that resistance of the interconnect influence only for perturbation parameter value.

After scaling we have:

$$
y = \frac{x}{d},\ \tau = \frac{t}{\sqrt{L_t C_t}},\ \tilde{v} = -\sqrt{\frac{C_t}{L_t}} v,\ \tilde{e} = \sqrt{\frac{C_t}{L_t}} e,\ \beta = \sqrt{\frac{C_t}{L_t}} R_s \tag{5}
$$

$$
\begin{aligned}
&\frac{\partial \tilde{v}}{\partial y} = \varepsilon i + \frac{\partial i}{\partial \tau}, \\
&\frac{\partial i}{\partial y} = \frac{\partial \tilde{v}}{\partial \tau},
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
&\tilde{e}(\tau) - \beta i(0,\tau) = -\tilde{v}(0,\tau), \\
&-i(1,\tau) = \frac{C_0}{C_t} \frac{\partial \tilde{v}(1,\tau)}{\partial \tau}
\end{aligned} \tag{7}
$$

where $C_t/C_0 = \alpha$.

The multiple scales method allows to compute [2] the lossly line output step response if the parameter ε fulfill condition (2). The first traveling voltage wave at the load capacitor can be expressed [2] as follow:

$$
\begin{aligned}
&v_s(d,t) = v_{01}(d,t) + \varepsilon v_{11}(d,t) = \\
&= \frac{E_0}{\beta+1}\left(A_2 \frac{t-T}{T} e^{-\frac{\alpha}{T}(t-T)} - B_2 \cdot \left(1 - e^{-\frac{\alpha}{T}(t-T)}\right) + C_2 \frac{t-T}{T} \right) \cdot \mathbf{1}(t-T),
\end{aligned} \tag{8}
$$

for time 0<t<3T

where:

$$
A_2 = \varepsilon \cdot e^{-0.5\varepsilon},\ C_2 = \varepsilon \frac{\beta}{\beta+1},\ B_2 = \left(\frac{\varepsilon}{\alpha} - 2\right) e^{-0.5\varepsilon} + \frac{\beta}{\beta+1}\frac{\varepsilon}{\alpha}.
$$

III. Sensitivity to RLC parameters

The step response can be sensitive even to small changes in the parameter values. The presented approach generates the closed form formula for output response (8), than we can calculate the sensitivity of the output response exact with the formula:

$$
S_v^{\lambda} = \frac{\lambda}{v}\frac{\partial v}{\partial \lambda} \tag{9}
$$

where λ is a parameter with respect to which sensitivity is calculated.

The sensitivity to parameters RLC are simply dependent on sensitivity of normalized parameters ε, β, α. The dependence can be written as[7]:

$$
S_{v_s}^{R} = \frac{1}{Z_0} v'_{s\varepsilon} \frac{R}{v_s} = \frac{\beta}{v_s} v'_{s\varepsilon} = S_{v_s}^{\varepsilon} \tag{10}
$$

$$
S_{v_s}^{L}(t) = -\frac{1}{2}\left(S_{v_s}^{\beta}(t) + S_{v_s}^{\varepsilon}(t)\right) - \frac{v'_{s\tilde{t}}}{v_s} \frac{t}{2\sqrt{LC}} \tag{11}
$$

$$
S_{v_s}^{C}(t) = S_{v_s}^{\alpha}(t) + \frac{1}{2}\left(S_{v_s}^{\beta}(t) + S_{v_s}^{\varepsilon}(t)\right) - \frac{v'_{s\tilde{t}}}{v_s} \frac{t}{2\sqrt{LC}} \tag{12}
$$

where v'_{sx} is a derivative of the voltage output response ($v_s(t)$) to parameter x.

The sensitivity to parameter α can be easily calculate and is given by [5]:

$$
S_{v_s}^{\alpha}(\tau) = \alpha \frac{\left(B_{2\alpha} - A_2 \tau^2 e^{-\alpha\tau} - B_2 \tau\right) e^{-\alpha\tau} - B_{2\alpha}}{A_2 \tau e^{-\alpha\tau} - B_2\left(1 - e^{-\alpha\tau}\right) + C_2 \tau} \tag{13}
$$

where $B_{2\alpha}$ is a derivative of B_2 with respect to α and $\tau = (t - T)/T$.

We can analyze the limit of $S_{v_s}^{\alpha}$ in $t = T^+$:

$$
S_{v_s}^{\alpha}\left(T^+\right) = \frac{\alpha}{v_s\left(T^+\right)} \frac{\partial v_s}{\partial \alpha}\bigg|_{t=T^+} = 1 \quad \left.\frac{\Delta v_s}{v_s\left(T^+\right)} \approx \frac{\Delta \alpha}{\alpha}\right| \tag{14}
$$

The obtained result is always 1. It means (9), that in the limit $t = T^+$ relative changes of α parameter cause the identical relative change in step response, independently to other parameters.

The sensitivity to parameter β is given by [5]:

$$
S_{v_s}^{\beta}(\tau) = \beta\left[\frac{-1}{\beta+1} + \frac{-B_{2\beta}\left(1 - e^{-\alpha\tau}\right) + C_{2\beta}\tau}{A_2 \tau e^{-\alpha\tau} - B_2\left(1 - e^{-\alpha\tau}\right) + C_2\tau}\right] \tag{15}
$$

where $B_{2\beta}$ is a derivative of B_2 to β and $C_{2\beta}$ is a derivative of C_2 to β and $\tau = (t - T)/T$.

Analyzing the limit of $S_{v_s}^{\beta}$ in $t = T^+$ we obtain that sensitivity to parameter β in that time point is:

$$
S_{v_s}^{\beta}\left(T^+\right) = \frac{\beta}{v_s\left(T^+\right)} \frac{\partial v_s}{\partial \beta}\bigg|_{t=T^+} = \frac{-\beta}{\beta+1} \quad \left.\frac{\left|\Delta v_s\right|}{v_s\left(T^+\right)} \approx \frac{\left|\Delta\beta\right|}{\beta}\frac{\beta}{\beta+1}\right| \tag{16}
$$

The relative changes of step response are proportional to relative changes of β with coefficient $\beta/(\beta+1)$ when the time $t=T^+$.

The sensitivity to parameter ε is given by [5]:

$$
S_{v_s}^{\varepsilon}(\tau) = \varepsilon\left[\frac{A_{2\varepsilon}\tau e^{-\alpha\tau} - B_{2\varepsilon}\left(1 - e^{-\alpha\tau}\right) + C_{2\varepsilon}\tau}{A_2 \tau e^{-\alpha\tau} - B_2\left(1 - e^{-\alpha\tau}\right) + C_2\tau}\right] \tag{17}
$$

where $A_{2\varepsilon}$ is a derivative of A_2 to ε, $B_{2\varepsilon}$ is a derivative of B_2 to ε and $C_{2\varepsilon}$ is a derivative of C_2 to ε and $\tau = (t-T)/T$.

Analyzing the limit of $S_{v_s}^{\varepsilon}$ in $t = T^+$ we obtain that sensitivity to parameter ε in that time point is:

$$S_{v_s}^{\varepsilon}\left(T^+\right) = \frac{\varepsilon}{v_s\left(T^+\right)}\frac{\partial v_s}{\partial \varepsilon}\bigg|_{t=T^+} = \frac{\varepsilon}{2} \qquad \frac{\left|\Delta v_s\right|}{v_s\left(T^+\right)} \approx \frac{\left|\Delta \varepsilon\right|}{\varepsilon}\frac{\varepsilon}{2}\bigg| \tag{18}$$

It means (9), that in time point $t = T^+$ relative changes of step response are proportional to relative changes of ε with coefficient $\varepsilon/2$.

The formula for sensitivity $S_{v_s}^{\varepsilon}$ is valid also for $S_{v_s}^{R}$ as mentioned earlier (10). The example of applying formula (18) to calculate the sensitivity of step response to resistance of the interconnect is presented in Fig.3. To calculate the sensitivity of inductance and capacitance of the interconnect we use the formulas (11)-(12). The examples are presented in Fig. 4 and Fig.5 respectively.

To present the results for the sensitivity of output response we consider the inverter-interconnect-inverter presented in Fig.1. We simulate the geometrical model in IE3D simulator to obtain the RLC parameters of the structure presented in Fig.2. The simulation result $R_t=124\Omega$, $C_t=0.2$pF, $L_t=12.4$nH, than the $\varepsilon=0.494$. We assume the modeled inverter output resistance $R_w=25\Omega$, and input capacitance $C_0=0.1$pF.

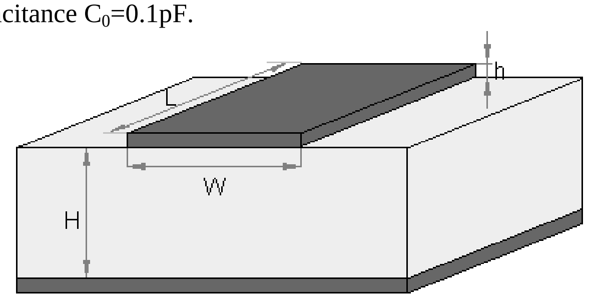

Fig. 2. Geometrical model used in the example. W= 2µm, H=300µm, h=1µm, l=5mm, $\varepsilon_{Si}=11.9$, $\sigma_{Si}=10000$S/m, $\sigma_{Cu}=2.73e+7$S/m

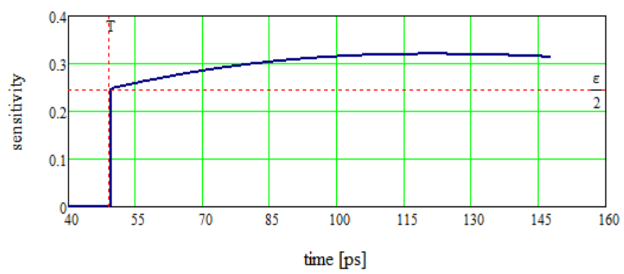

Fig. 2. Sensitivity of step response to *R* for an interconnect for parameters presented in Fig.2

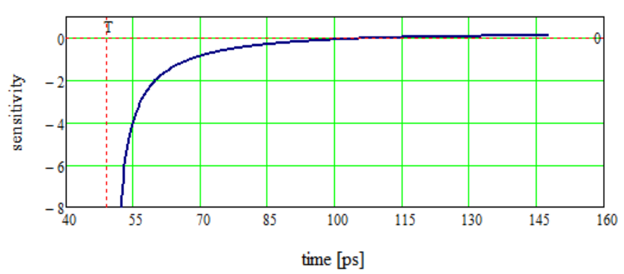

Fig. 3. Sensitivity of step response to *L* for an interconnect for parameters presented in Fig.2

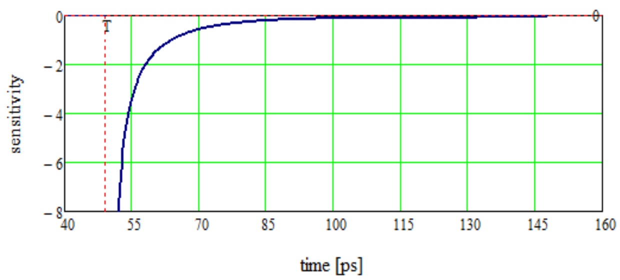

Fig. 4. Sensitivity of step response to *C* for an interconnect for parameters presented in Fig.2

IV. Step response deviation to parameter variations

Sensitivity coefficients: (10), (11), (12) are functions of time t and parameters $\alpha, \beta, \varepsilon$. There is a possibility to estimate deviation of step response (8) with respect to parameter change in some range. We assume the parameter deviation $\lambda=\lambda_0 \pm \Delta\lambda$. Expanding step response in Taylor series with respect to parameter λ and taking into account only first terms we obtain:

$$\begin{aligned} v_+(t) &= v_s(t,\lambda_+) \approx v_s(t,\lambda_0) + \frac{\partial v_s(t,\lambda)}{\partial \lambda}\bigg|_{\lambda=\lambda_0} \Delta\lambda \\ v_-(t) &= v_s(t,\lambda_-) \approx v_s(t,\lambda_0) - \frac{\partial v_s(t,\lambda)}{\partial \lambda}\bigg|_{\lambda=\lambda_0} \Delta\lambda. \end{aligned} \tag{19}$$

or

$$v_+(t) \approx v_s(t,\lambda_0)\left(1 + S_v^{\lambda}(t,\lambda)\cdot\frac{\Delta\lambda}{\lambda}\right) \tag{20}$$

$$v_-(t) \approx v_s(t,\lambda_0)\left(1 - S_v^{\lambda}(t,\lambda)\cdot\frac{\Delta\lambda}{\lambda}\right). \tag{21}$$

Step response deviation applied to system inverter - interconnect - inverter with 5% parameters deviation are presented in Fig.5-7. The RLC values for simulation are taken from the structure presented in Fig.2. For the comparison there is also presented in Fig.8. the deviation of step response for 20% capacitance deviation.

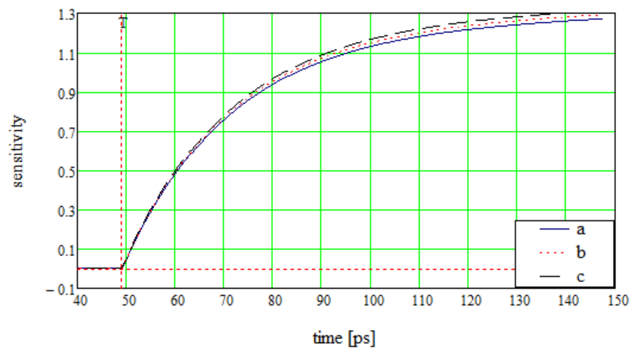

Fig. 5. Step response with R variation in sensitivity of 5% for an interconnect for parameters presented in Fig.2 a) (20), b)v(t), c) (21),

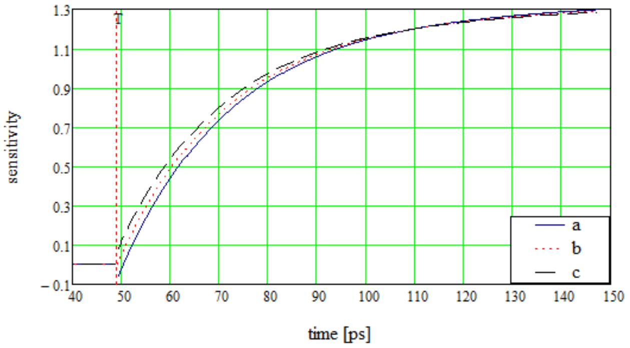

Fig. 6. Step response with L variation in sensitivity of 5% for an interconnect for parameters presented in Fig.2 a) (20), b)v(t), c) (21),

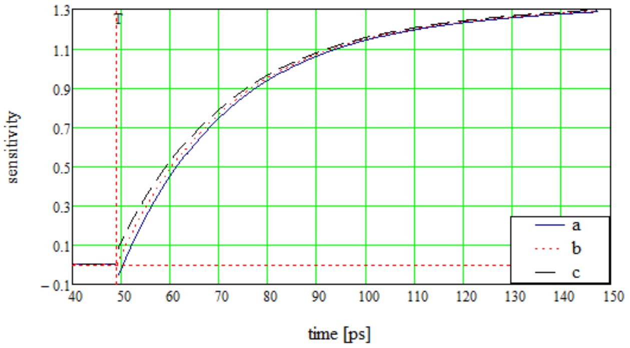

Fig. 7. Step response with C variation in sensitivity of 5% for an interconnect for parameters presented in Fig. 2 a) (20), b)v(t), c) (21),

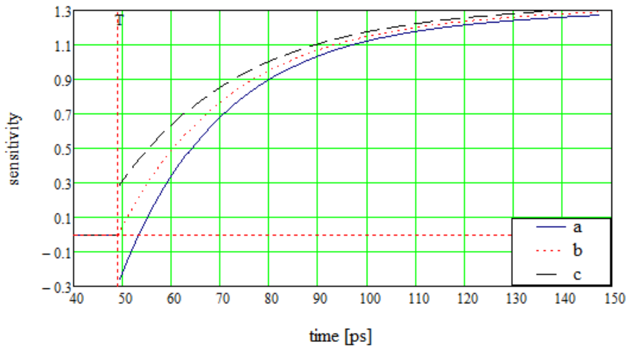

Fig. 8. Step response with C variation in sensitivity of 20% for an interconnect for parameters presented in Fig.2 a) (20), b)v(t), c) (21),

The presented example shows, that the *L* and *C* variations have the most impact in the first moment of rising signal. The reason is, that the *L* or *C* change cause the time delay change. Then the signal is, for $t=T$, most sensitive for these parameters. Changing the resistance of the interconnect has the bigger impact on the steady state of the signal and the rising of the signal is almost insensitive on it.

In the presented example the 50% threshold crossing time $t_{\rho 50\%}$=60.04ps. The deviation 5% in capacitance value causes that the time can be in the range: 59,31ps<t<61,44ps. It means that we can estimate the time error as:

$$\delta t_{\rho 50\%} = \frac{t_{\rho 50\%+} - t_{\rho 50\%-}}{t_{\rho 50\%}} 100\% \tag{22}$$

Then for presented above example the time error will be 3,5%.

Analyzing the summarized impact of RLC parameters we calculate sensitivity as:

$$S_{vs}(t) = \left|S_{vs}^{C}(t)\right| + \left|S_{vs}^{L}(t)\right| + \left|S_{vs}^{R}(t)\right| \tag{23}$$

Applying the formula to the voltage response and calculating the deviation of the voltage on parameter changes we obtain the results presented in Fig.8. similarly as earlier (20),(21). The time error (22) in that case is 4.2%.

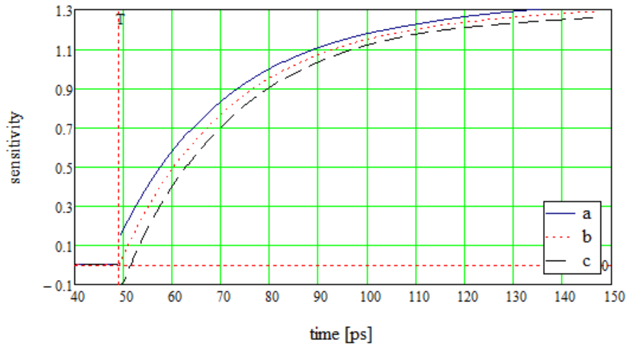

Fig. 8. Step response with RLC variation in sensitivity (23) of 5% for an interconnect for parameters presented in Fig.2 a) (20), b)v(t), c) (21),

V. Conclusions

Sensitivity analysis allows to estimate the impact of small parameter changes on signal deviation. The proposed formulas are fast and efficient way to obtain sensitivity of signal to interconnect parameter changes. In the article the example of signal interconnect is presented. The RLC parameters was obtained with IE3D simulator. Obtained parameters was used to estimate the deviation of step response using the closed form formulas for sensitivity. The method can be used instead of Statistic Montecarlo techniques which are commonly used tools to determine the bounds of the system response to parameter deviations.

References

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